

On multidimensional solutions in the Einstein-Gauss-Bonnet model with a cosmological term

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Abstract

A D -dimensional gravitational model with Gauss-Bonnet and cosmological term Λ is considered. When ansatz with diagonal cosmological metrics is adopted, we overview recent solutions for $\Lambda = 0$ and find new examples of solutions for $\Lambda \neq 0$ and $D = 8$ with exponential dependence of scale factors which describe an expansion of “our” 3-dimensional factor-space and contraction of 4-dimensional internal space.

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1 Introduction

Here we consider D -dimensional gravitational model with the Gauss-Bonnet term. The action reads

$$S = \int_M d^D z \sqrt{|g|} \{ \alpha_1 (R[g] - 2\Lambda) + \alpha_2 \mathcal{L}_2[g] \}, \quad (1.1)$$

where $g = g_{MN} dz^M \otimes dz^N$ is the metric defined on the manifold M , $\dim M = D$, $|g| = |\det(g_{MN})|$ and

$$\mathcal{L}_2 = R_{MNPQ} R^{MNPQ} - 4R_{MN} R^{MN} + R^2 \quad (1.2)$$

is the standard Gauss-Bonnet term. Here α_1 and α_2 are non-zero constants.

Earlier the appearance of the Gauss-Bonnet term was motivated by string theory [1, 2, 3, 4, 5].

At present, the (so-called) Einstein-Gauss-Bonnet (EGB) gravitational model and its modifications are intensively used in cosmology, see [6] (for $D = 4$), [7, 8, 9, 10, 11, 12, 13, 14, 15] and references therein, e.g. for explanation of accelerating expansion of the Universe following from supernovae (type Ia) observational data [16, 17, 18]. Certain exact solutions in multidimensional EGB cosmology were obtained in [7]-[15] and some other papers.

Here we are dealing with the cosmological type solutions with diagonal metrics (of Bianchi-I-like type) governed by n scale factors depending upon one variable, where $n > 3$. Moreover, we restrict ourselves by the solutions with exponential dependence of scale factors. We present new examples of exact solutions in dimension $D = 8$ which describe an exponential expansion of 3-dimensional factor-space and contraction of 4-dimensional internal space.

2 The Cosmological Model

Here we consider the manifold

$$M = \mathbb{R} \times \mathbb{R}^n \quad (2.1)$$

with the metric

$$g = -dt \otimes dt + \sum_{i=1}^n e^{2\beta^i(t)} dy^i \otimes dy^i, \quad (2.2)$$

where $\beta^i(t)$ are smooth functions, $i = 1, \dots, n$.

We introduce ‘‘Hubble-like’’ variables $h^i = d\beta^i/dt$. The equations of motion for the action (1.1) read as follows

$$\alpha_1 (G_{ij} h^i h^j + 2\Lambda) - \alpha_2 G_{ijkl} h^i h^j h^k h^l = 0, \quad (2.3)$$

$$\left[2\alpha_1 G_{ij} h^j - \frac{4}{3} \alpha_2 G_{ijkl} h^j h^k h^l \right] \sum_{i=1}^n h^i + \frac{d}{dt} \left[2\alpha_1 G_{ij} h^j - \frac{4}{3} \alpha_2 G_{ijkl} h^j h^k h^l \right] - L = 0, \quad (2.4)$$

$i = 1, \dots, n$, where

$$L = \alpha_1(G_{ij}h^i h^j + 2\Lambda) - \frac{1}{3}\alpha_2 G_{ijkl}h^i h^j h^k h^l. \quad (2.5)$$

Here

$$G_{ij} = \delta_{ij} - 1, \quad (2.6)$$

$$G_{ijkl} = G_{ij}G_{ik}G_{il}G_{jk}G_{jl}G_{kl} \quad (2.7)$$

are respectively the components of two metrics on \mathbb{R}^n [19, 20]. The first one is the well-known “minisupermetric” - 2-metric of pseudo-Euclidean signature and the second one is the Finslerian 4-metric.

Due to (2.3)

$$L = \frac{2}{3}\alpha_1(G_{ij}h^i h^j - 4\Lambda). \quad (2.8)$$

In this paper we deal with the following solutions to equations (2.3) and (2.4)

$$h^i(t) = v^i, \quad (2.9)$$

with constant v^i , which corresponding to the solutions

$$\beta^i = v^i t + \beta_0^i, \quad (2.10)$$

where β_0^i are constants, $i = 1, \dots, n$.

In this case we obtain the metric (2.2) with the exponential dependence of scale factors

$$g = -dt \otimes dt + \sum_{i=1}^n B_i e^{2v^i t} dy^i \otimes dy^i, \quad (2.11)$$

where $B_i > 0$ are arbitrary constants.

For the fixed point $v = (v^i)$ we have the set of polynomial equations

$$G_{ij}v^i v^j + 2\Lambda - \alpha G_{ijkl}v^i v^j v^k v^l = 0, \quad (2.12)$$

$$\left[2G_{ij}v^j - \frac{4}{3}\alpha G_{ijkl}v^j v^k v^l \right] \sum_{i=1}^n v^i - \frac{2}{3}G_{ij}v^i v^j + \frac{8}{3}\Lambda = 0, \quad (2.13)$$

$i = 1, \dots, n$, where $\alpha = \alpha_2/\alpha_1$. For $n > 3$ this is a set of forth-order polynomial equations.

For $\Lambda = 0$ and $n > 3$ the set of equations (2.12) and (2.13) has an isotropic solution $v^1 = \dots = v^n = H$, only if $\alpha < 0$ [19, 20] $H = \pm 1/\sqrt{|\alpha|(n-2)(n-3)}$. This solution was generalized in [15] to the case $\Lambda \neq 0$.

It was shown in [19, 20] that there are no more than three different numbers among v^1, \dots, v^n when $\Lambda = 0$. This is valid also for $\Lambda \neq 0$.

3 Examples of Cosmological Solutions

In this section we consider some solutions to the set of equations (2.12), (2.13) of the following form

$$v = (H, \dots, H, h, \dots, h). \quad (3.1)$$

where H the “Hubble-like” parameter corresponding to m -dimensional isotropic subspace with $m > 3$ and h is the “Hubble-like” parameter corresponding to l -dimensional isotropic subspace, $l > 2$.

These solutions should satisfy the following conditions: $H > 0$, $h < 0$. The first inequality $H > 0$ is necessary for a description of accelerated expansion of 3-dimensional subspace, which may describe our Universe, while the second inequality $h < 0$ is necessary for contraction of internal space volume.

3.1 Polynomial equations

According to our ansatz (3.1), we have m dimensions expanding with the Hubble parameter $H > 0$ and l dimensions contracting with the “Hubble-like” parameter $h < 0$. The set of polynomial equations (2.12), (2.13) reads

$$\begin{aligned} & H^2(m - m^2) + h^2(l - l^2) - 2mlHh \\ & -\alpha(H^4m(m-1)(m-2)(m-3) + h^4l(l-1)(l-2)(l-3) \\ & + 4H^3hm(m-1)(m-2)l + 4h^3Hl(l-1)(l-2)m \\ & + 6H^2h^2m(m-1)l(l-1)) + 2\Lambda = 0, \end{aligned} \quad (3.2)$$

$$\begin{aligned} & m(1-m)H^2 - (1/2)lh^2(1+2l) + 2lHh((3/4)-m) \\ & -\alpha(H^4m(m-1)(m-2)(m-3) + H^3hl(m-1)(m-2)(4m-3) \\ & + 3H^2h^2l(m-1)(2lm-2l-m) \\ & + Hh^3l(l-1)(4lm-3l-2m) + h^4l^2(l-1)(l-2)) + 2\Lambda = 0, \end{aligned} \quad (3.3)$$

$$\begin{aligned} & l(1-l)h^2 - (1/2)mH^2(1+2m) + 2mHh((3/4)-l) \\ & -\alpha(h^4l(l-1)(l-2)(l-3) + h^3Hm(l-1)(l-2)(4l-3) \\ & + 3h^2H^2m(l-1)(2lm-2m-l) \\ & + hH^3m(m-1)(4lm-3m-2l) + H^4m^2(m-1)(m-2)) + 2\Lambda = 0. \end{aligned} \quad (3.4)$$

We put $\alpha = \pm 1$ and denote $\Lambda = \lambda$, keeping in mind the general α -dependent form of solution

$$H(\alpha) = H|\alpha|^{-1/2}, \quad h(\alpha) = h|\alpha|^{-1/2} \quad \Lambda = \lambda|\alpha|^{-1}. \quad (3.5)$$

3.2 Solutions with $\Lambda = 0$

Let $\Lambda = 0$ and $\alpha = 1$. It was shown in [21] that, for $m = 9$ there exists an infinite series of cosmological solutions with $l = 3000, 3001, \dots$, any of which describes an accelerated expansion of the 3-dimensional factor space with sufficiently small value of the variation of the effective gravitational constant G obeying the observational restrictions [22], see also [23]. This variation may be arbitrary small for a big enough value of l . We remind that the effective gravitational constant G is proportional to the inverse volume scale factor of the internal space, see [24, 25, 26, 27, 28] and references therein.

For $m = 11$ and $l = 16$ it was found in [21] a solution with

$$H = \frac{1}{\sqrt{15}}, \quad h = -\frac{1}{2\sqrt{15}}, \quad (3.6)$$

which describe a zero variation of effective cosmological constant G .

Another solution of such type which was found in [21] appears for $m = 15$ and $l = 6$ with

$$H = \frac{1}{6}, \quad h = -\frac{1}{3}. \quad (3.7)$$

3.3 Solutions with $\Lambda \neq 0$

Here we present several new cosmological solutions for $\Lambda \neq 0$, $\alpha = 1$, $m = 3$ and $l = 4$.

The first solution takes place for $\lambda = 3/16$:

$$H = \frac{1}{4}\sqrt{2}, \quad h = -\frac{1}{4}\sqrt{2}. \quad (3.8)$$

The second one is valid for $\lambda = 13/48$:

$$H = \frac{1}{4}\sqrt{6}, \quad h = -\frac{1}{12}\sqrt{6}. \quad (3.9)$$

The third one

$$H = \frac{2}{29}\sqrt{29}, \quad h = -\frac{3}{58}\sqrt{29}. \quad (3.10)$$

corresponds to $\lambda = 21/116$.

Any of these solutions describe accelerated expansion of 3-dimensional factor space and contraction of internal space. All of these solutions take place for fixed positive values of λ (see (3.5)). There exist also examples of solutions with negative cosmological constant $\lambda = -21/80$:

$$H_{\pm} = \frac{1}{60320}(248 \pm 32\sqrt{30})\sqrt{68150 \mp 9280\sqrt{30}}, \quad (3.11)$$

$$h_{\pm} = -\frac{1}{580}\sqrt{68150 \mp 9280\sqrt{30}}. \quad (3.12)$$

These solutions obey inequalities $H_{\pm} > 0$ and $h_{\pm} < 0$.

4 Conclusions

We have considered the D -dimensional Einstein-Gauss-Bonnet (EGB) model with with Λ -term. By using the ansatz with diagonal cosmological type metrics, we have found new solutions with exponential dependence of scale factors with respect to synchronous time variable t in dimension $D = 1 + 3 + 4$. Any of these solutions describes an exponential expansion of “our” 3-dimensional factor-space with the Hubble parameter $H > 0$ and exponential contraction of $4d$ internal space. An open question arising here is to find solutions with $\Lambda \neq 0$ which obey the observational constraints on the temporal variation of the effective gravitational constant G . This question will be addressed in a separate publication.

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